

# Cooperation enhanced by the difference between interaction and learning neighborhoods for evolutionary spatial prisoner's dilemma games

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We study an evolutionary prisoner's dilemma game with two layered graphs, where the lower layer is the physical infrastructure on which the interactions are taking place and the upper layer represents the connections for the strategy adoption (learning) mechanism. This system is investigated by means of Monte Carlo simulations and an extended pair-approximation method. We consider the average density of cooperators in the stationary state for a fixed interaction graph, while varying the number of edges in the learning graph. According to the Monte Carlo simulations, the cooperation is modified substantially in a way resembling a coherence-resonance-like behavior when the number of learning edges is increased. This behavior is reproduced by the analytical results.

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## I. INTRODUCTION

Cooperation can be found in many places in the realistic world, from biological systems to economic and social systems [1]. An altruistic action, which benefits others at the expense of one's own investment, appears to contradict our understanding of natural selection controlled by selfish individual behaviors. Thus understanding the conditions for the emergence and maintenance of cooperative behavior among unrelated and selfish individuals becomes a central issue in evolutionary biology [2]. In the investigation of this problem the most popular framework is game theory together with its extensions involving evolutionary context [3, 4]. The prisoner's dilemma (PD), a two-person game in which the players can choose either cooperation ( $C$ ) or defection ( $D$ ), is a common paradigm for studying the evolution of cooperation [5, 6]. In the traditional version of the PD game, two interacting players are offered a certain payoff, the reward  $R$ , for mutual cooperation and a lower payoff, the punishment  $P$ , for mutual defection. If one player cooperates while the other defects, then the cooperator gets the lowest sucker's payoff  $S$ , while the defector gains the highest payoff, the temptation to defect  $T$ . Thus we obtain  $T > R > P > S$ . It is easy to see that defection is the better choice irrespective of the opponent's selection. For this reason, defection is the only evolutionarily stable strategy in fully mixed populations of  $C$  and  $D$  strategies [3].

Since cooperation is abundant and robust in nature, considerable efforts have been concentrated on exploration of the origin and persistence of cooperation. During the last decades, five rules, namely, kin selection [7], direct reciprocity [5], indirect reciprocity [8], network (or spatial) reciprocity [9, 10, 11], and group selection [12], have been found to benefit the evolution of cooperation in biological and ecological systems as well as within human societies (for a recent review, see [13] and references therein). In realistic sys-

tems, most interactions among elements are spatially localized, which makes spatial or graph models more meaningful. Unlike the other four rules, spatial games (i.e., network reciprocity) can lead to cooperation in the absence of any strategic complexity [9, 10, 11, 14] (for a recent review of evolutionary games on graphs, see [15]). In spatial evolutionary PD games, the cooperators can survive by forming large compact clusters, which minimize the exploitation by defectors. Along the boundary, cooperators can outweigh their losses against defectors by gains from interactions within the cluster [14, 16, 17].

In spatial models [10, 11, 14, 16, 17], the players occupying the vertices of a graph can follow one of the two pure strategies ( $C$  or  $D$ ), and collect payoffs from their neighbors by playing PD games. Sometimes the players are allowed to modify their strategies according to an evolutionary rule dependent on the local payoff distribution. To describe real systems we can introduce two different graphs [13]. The "interaction graph" determines who plays with whom. The "replacement graph" (or learning graph) determines who competes with whom for reproduction, which can be genetic or cultural. To our knowledge, in most of the existing works the interaction and replacement graphs are assumed to be identical. The different roles of these graphs raises a natural question: How is cooperation affected when the interaction and replacement graphs are different? Ifti *et al.* [18] have studied the continuous PD game when the interaction neighborhood (IN) and learning neighborhood (LN) are different. In the lattice topology, it was observed that when the neighborhood sizes for "interacting" and "learning" differ by more than 0.5, cooperation is not sustainable [18]. Now we wish to study what happens if the players can follow only one of the two pure strategies and the LN for the individuals is inhomogeneous.

In this paper, we address these problems by considering an evolutionary PD game on two layered graphs. The lower layer is the physical infrastructure on which the interactions are taking place (interaction layer), and the upper layer represents the information flows (learning or imitation layer). For the sake of simplicity, we study the case where the lower interaction layer is a square lattice. Generally, one can expect that the size of the LN is larger than that of the IN, which can

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be understood as follows. After each round of the game, not only do the interacting players exchange information about their own payoffs and strategies, they also share information about their neighbors and their neighbors' neighbors. To explore the influence of the difference between the interaction and learning graphs on the evolution of cooperation, we keep the IN fixed and vary the size of the LN. In what follows two types of models are systematically investigated. In the first case (model I), we simply increase the size of the LN for all the players at the same level. In the second case (model II), we endow the players with heterogeneous abilities to obtain information, i.e., some players have a larger size of LN than others.

## II. MODEL

We consider the PD game with pure strategies: either  $C$  or  $D$ . On the interaction layer (a square lattice), each player plays PD games with its four neighbors and collects a payoff determined by the strategy-dependent payoff. The total payoff of a certain player is the sum over all interactions. We assume that a cooperator pays a cost  $c$  for another individual to receive a benefit  $b$  ( $b > c$ ), and a defector pays no cost and does not distribute any benefits. Thus the reward for mutual cooperation is  $R = b - c$ , the sucker's payoff  $S = -c$ , the punishment for mutual defection is  $P = 0$ , and the temptation to defect is  $T = b$ . Following [17], the payoffs are rescaled such that  $R = 1$ ,  $T = 1 + r$ ,  $S = -r$ , and  $P = 0$ , where  $r = c/(b - c)$  denotes the ratio of the costs of cooperation to the net benefits of cooperation.

After each round of the game, the players are allowed to inspect their learning neighbors' payoffs and strategies, and, according to the comparison, determine which of their strategies to adopt in the next round. Following previous studies [14, 15, 17, 19, 20, 21, 22], the evolution of the present system is governed by the adoption of strategy by a randomly chosen player  $i$  and one of its learning neighbors  $j$ , namely, the player  $i$  will adopt the learning neighbor's strategy with a probability dependent on the payoff difference ( $U_i - U_j$ ) as

$$W = \frac{1}{1 + \exp[(U_i - U_j)/\kappa]}, \quad (1)$$

where  $\kappa$  characterizes the noise introduced to permit irrational choices.  $\kappa = 0$  and  $\kappa \rightarrow \infty$  denote the completely deterministic and completely random selection of the neighbor's strategy, respectively, while for any finite positive values  $\kappa$  incorporates the uncertainties in the strategy adoption, i.e., the *better* one's strategy is readily adopted, but there is a small probability to select the *worse* one's. The effect of noise  $\kappa$  on the stationary density of cooperators in the spatial PD game has been studied in detail in Refs. [20, 22]. Since this issue goes beyond the purpose of the present work, in all our following studies, we simply fix the value of  $\kappa$  to be  $\kappa = 0.1$ .

In both models I and II, the lower interaction graph is a square lattice with periodic boundary conditions and of size  $N = 200 \times 200$ . For model I, we denote the size of the LN of the players by  $d$ , where  $d = 1, 2, \dots$  indicate, respectively,

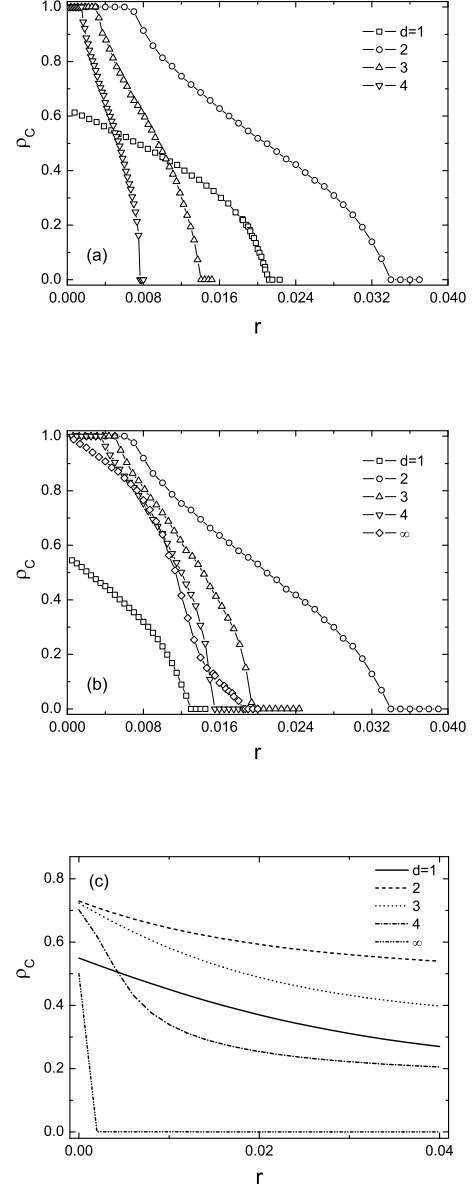


FIG. 1: Average density of cooperators,  $\rho_C$ , as a function of  $r$  for different sizes of the LN on square lattices with asynchronous (a) and synchronous (b) strategy-updating. Predictions of  $\rho_C$  by the pair approximation are shown in (c). The cases  $d = 1, 2, 3, 4$  correspond to, respectively, the conditions that the learning neighborhoods of the players include their nearest neighbors, nearest and next-nearest neighbors, and so on, while  $d = \infty$  means that each player can learn from the whole population.

that each player can learn (or get payoff and strategy information after each round) from their nearest neighbors, nearest and next-nearest neighbors, and so on. For model II, the upper learning graph is a scale free network embedded on the underlying square lattice, which can be constructed according to the following steps, associated with the lattice embed-

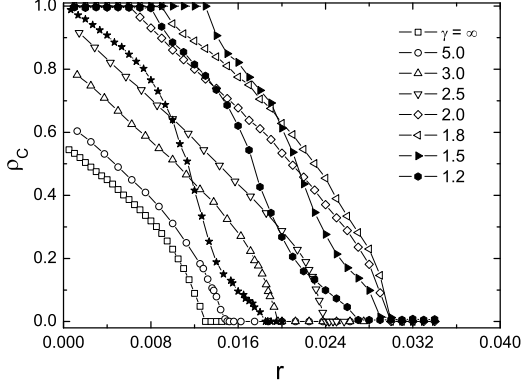


FIG. 2: Average density of cooperators,  $\rho_C$ , as a function of  $r$  on square lattices with synchronous strategy-updating, where the learning networks are the LESFNs built on the underlying square lattices with different decay exponents  $\gamma$ . For the sake of comparison, the case that each player can learn from the whole population is also shown by solid stars.

ded scale-free network (LESFN) model [23]: For each site of the underlying interaction graph, a prescribed degree  $k$  is assigned taken from a scale-free distribution  $P(k) \sim k^{-\gamma}$ ,  $k \in [4, N]$ . A node (say  $i$ , with degree  $k_i$ ) is picked out randomly and connected to its closest neighbors until its degree quota  $k_i$  is realized or until all sites up to a distance  $r(k_i) = \min(A\sqrt{k_i}, \sqrt{N})$  have been explored, where  $A$  is the territory parameter [23] (in the present work, we set  $A = 10$ ). Duplicate connections are avoided. This process is repeated for all sites of the underlying lattice [24].

### III. RESULTS

First, we study the two models by Monte Carlo (MC) simulations started from a random initial distribution of  $C$  and  $D$  strategies. By varying the value of  $r$ , both asynchronous and synchronous strategy-updating are implemented for model I, and only synchronous for model II. The total sampling times are 16000 MC steps and up to 24000 for model II when  $\gamma < 2.0$ . The stationary state is characterized by the average density of cooperators  $\rho_C$  calculated by averaging over the last 4000 steps when the values of  $d$  and  $\gamma$  are varied systematically. All the simulation data shown in Figs. 1, 2, and 3 result from an average over either ten realizations of independent initial strategy configurations (for model I) or ten realizations of the learning graphs (for model II).

Let us first discuss the MC results obtained for model I. The dependence of  $\rho_C$  on  $r$  in the stationary state for different sizes of LN,  $d$ , is illustrated in Figs. 1(a) and 1(b). For  $d = 1$ , i.e., when the IN and LN are identical, and with asynchronous strategy updating, we recover the results of [17]: cooperators persist at substantial levels if  $r$  is sufficiently small [Fig. 1(a)]. Synchronous strategy updating gives rise to a smaller thresh-

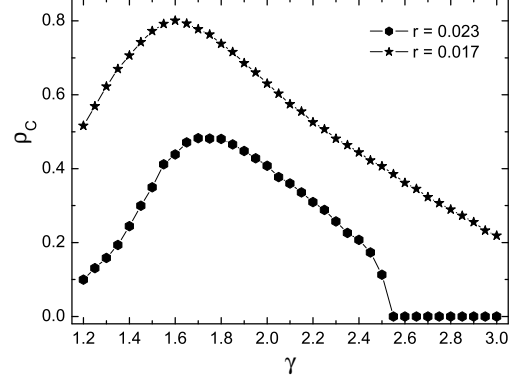


FIG. 3: Average density of cooperators,  $\rho_C$ , as a function of the decay exponent  $\gamma$  of the LESFNs for two special values of  $r = 0.017$  and  $0.023$ .

old of  $r_c$ , beyond which cooperators vanish [Fig. 1(b)]. It is interesting that for  $d = 2$ , i.e., besides its nearest neighbors a player can also learn from its next-nearest neighbors, both asynchronous and synchronous strategy updating lead to qualitatively as well as quantitatively the same stationary density of  $\rho_C$ . For even larger sizes of  $d = 3$  and  $4$ , though the qualitative behaviors are similar, their quantitative properties are distinct [somewhat greater values of the threshold  $r_c$  in Fig. 1(b) for synchronous dynamics]. In particular, for  $d \rightarrow \infty$ , which corresponds to the case that each player can learn from the whole population, cooperators cannot persist in the system for any finite positive values of  $r$  when updating asynchronously, whereas they can maintain at considerable levels if  $r$  is very small when updating synchronously.

In addition to the above points, it is worth pointing out that, when the LN is larger than the IN, e.g.,  $d = 2, 3$ , and  $4$ , there arise two absorbing states (all  $C$  and all  $D$ , respectively) separated by an active state (coexistence of  $C$  and  $D$ ) over the range of  $r$ , i.e., cooperators can “wipe out” defectors or dominate in the system if the players are allowed to get payoff and strategy information from neighbors further away than just interacting neighbors only. This is to say, cooperation is promoted due to the difference between the IN and LN. Figures 1(a) and 1(b) illustrate clearly the remarkable enhancement appearing in the case of  $d = 2$  for both synchronous and asynchronous dynamics. For synchronous strategy updating, however, as long as the size of the LN is larger than that of the IN, the cooperative behavior is always enhanced to some extent as compared to the case of  $d = 1$ . For asynchronous strategy updating, however, the promotion of cooperation is only realized in a very small range of  $r$  when  $d > 2$ , and this range decreases with increasing  $d$  and vanishes in the limit of  $d \rightarrow \infty$ .

The mean-field approximation predicts  $\rho_C = 0$  for any values of  $0 < r < 1$  [15, 17, 20]. The nonzero values of  $\rho_C$  (dependent on  $r$ ) cannot be described by the mean-field approach. To characterize the evolution of  $\rho_C$ , the more sophis-

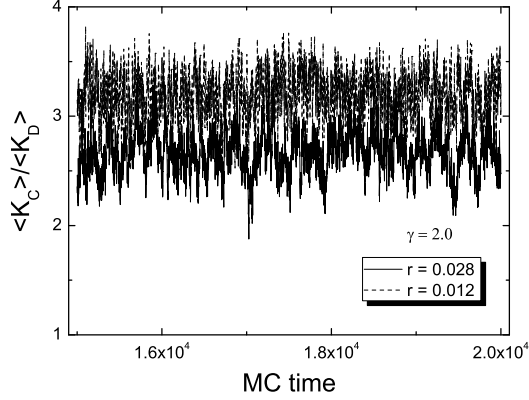


FIG. 4: Typical time series of the ratio of average learning degree of cooperators and defectors,  $\langle K_C \rangle / \langle K_D \rangle$ , for two special values of  $r = 0.012$  and  $0.028$  in the stationary state. The upper LESFN has a decay exponent  $\gamma = 2.0$ .

ticated pair approximation provides an analytically accessible way to determine the corrections from spatial structural correlation of the players. Instead of the equilibrium density of  $C$  and  $D$ , the pair approximation considers the frequency of strategy pairs  $C$ - $D$  (see the Appendix of Refs. [15, 17] or the Supplementary Information of Ref. [16] for details). However, these existing methods are prepared for the condition where the interaction and learning graphs are identical. When these two graphs are different, we should make some modifications to the original approach.

In the present work, we use an extended pair-approximation (see the Appendix) to calculate the density  $\rho_C$  by varying the values of  $r$  and  $d$  for model I. The results are shown in Fig. 1(c). The extended pair-approximation correctly predicts the tendencies of the evolution of  $\rho_C$ , especially for  $d = 2$ , but significantly underestimates the benefits of spatial structural effects and the larger size of the LN (than the IN) at low  $r$ , whereas it overestimates those at high  $r$ . Despite this point, it verifies the above result obtained by MC simulation, i.e., the most remarkable enhancement of cooperation takes place at  $d = 2$ . For  $d = 3$ , it fits the synchronous results better than the asynchronous ones [according to the magnitude relationship between the curve for  $d = 3$  and that for  $d = 1$  in Fig. 1(a)-Fig. 1(c)], despite the fact that the pair-approximation is based on the assumption of continuous time, and hence on asynchronous updating. In particular, for  $d = 4$ , it correctly predicts the occurrence of an intersection with the curve for  $d = 1$  as previously found by MC simulation in Fig. 1(a). For  $d \rightarrow \infty$ , it once again correctly forecasts the extinction of cooperators ( $\rho_C = 1/2$  for  $r = 0$ , and  $\rho_C = 0$  for other any finite positive values of  $r$ ).

We now focus our attention on the influence of the heterogeneous LN on the evolution of cooperation. The MC results obtained for model II with different values of  $\gamma$  are summarized in Fig. 2. The case  $\gamma \rightarrow \infty$  is equivalent to the case

$d = 1$  studied in model I. With decreasing  $\gamma$  (yielding an increase in the average degree of the LN), the cooperative level increases gradually until  $\gamma \approx 1.7 \pm 0.2$ , where  $\rho_C$  reaches its maximum, and then it gradually decreases as  $\gamma$  goes to zero. In finite size systems for vanishing  $\gamma$  in model II, the evolutionary results are expected to tend toward the (unattainable) case of  $d \rightarrow \infty$  in model I, since on average the players have more and more learning neighbors. For the sake of clarity, in Fig. 2 we also show the result for  $d \rightarrow \infty$  obtained in model I by solid stars.

Note that, just as was found in model I, if the LN and IN are different, then cooperation is promoted, and the maximum enhancement is achieved at a moderate level of the available information of the LN. Too little information as well as too much information favors defection. To support this point, we have also studied the density of cooperators as a function of the size of the LN of the players (characterized by  $\gamma$ ) for two special values of  $r = 0.017$  and  $0.023$ . The MC results are plotted in Fig. 3. We can clearly observe that there exactly arise peak values of  $\rho_C$  in the middle range of  $\gamma$  for both  $r$ , analogously to the so-called coherence resonance [25]. In recent research work, many mechanisms are described that can lead to this coherence resonance phenomenon in studying the PD game. For example: in Ref. [25] additive noise on the classical replicator dynamics can enhance the average payoff of the population in a resonance-like manner; By introducing random disorder in the payoff matrix, Perc [26] found a resonance-like behavior of the density of the cooperators which reaches its maximum at an intermediate disorder. On static complex networks, Tang *et al.* [27] obtained the result that maximum cooperation level occurs at intermediate average degree. Ren *et al.* [28] studied the effects of both topological randomness in individual relationships and dynamical randomness in decision making on the evolution of cooperation, and found that there exists an optimal moderate level of randomness, which can induce the highest level of cooperation. Our result presented here, i.e., enhancing cooperation by increase of the LN, which resembles a coherence-resonance-like behavior, provides a different example of this dynamical phenomenon. It will enrich our knowledge of the evolution of cooperation in nature.

More recently, Ohtsuki *et al.* [29] studied the evolution of cooperation in the evolutionary spatial PD game, wherein the interaction graph and replacement (or learning) graph are separated. They considered three different update rules for evolutionary dynamics: birth-death, death-birth, and imitation [29]. By both analytical treatment and computer simulations, they found that under death-birth and imitation updating, the optimum population structure for cooperators is given by maximum overlap between the interaction and the replacement graph, i.e., whenever the two graphs are identical [29]. Any existing difference between these two graphs will benefit defectors. This result holds for weak-selection (which means that the payoffs obtained by the individuals from the game have a slight contribution to their fitness) and large population size. The “imitation” updating in [29] is implemented as follows. A random individual is chosen to update its strategy; it will either stay with its own strategy or imitate one of the

neighbors' strategies proportional to its fitness. In fact, from this point of view, the update mechanism (or evolutionary dynamics) of our model, Eq. (1), can also be regarded as imitation, where the fitness of each individual is determined by an exponential function of its payoff obtained from the game,  $e^{U/\kappa}$  (Whenever updating the state of the population, one by one the focal individual and a randomly chosen neighbor from its LN compete for reproduction proportional to their fitness according to this function.) However, we obtain remarkably different result as compared to [29], i.e., in our model, the difference between the IN and LN can favor essentially cooperators over defectors (especially for the case of synchronous updating). Since the evolutionary outcomes are dependent on the updating rules, and there are many possible updating dynamics on graphs, we think the detailed evolutionary rules give rise to this different result in contrast to that of [29]. In addition, in the present model the fitness of the individuals is closely related to their payoff, which can be regarded as strong selection, while the result in [29] is obtained in the limit of weak selection. Thus our present results enrich our knowledge of the evolution of cooperation in the PD game when the IN and LN are separated.

Since in model II the players possess an inhomogeneous LN, we would like to investigate the effect of this heterogeneity on the players' strategy selection. In Fig. 4 we display the typical stationary-state time series of the ratio of the average learning degrees of cooperators and defectors (calculated by the total number of neighbors learning of a certain strategy divided by the total number of the players adopting this strategy),  $\langle K_C \rangle / \langle K_D \rangle$ , for two special values of  $r = 0.012$  and  $0.028$  (cooperators and defectors dominate in the two cases, respectively). The upper LESFN has a decay exponent  $\gamma = 2.0$ . We can observe that in the stationary state the average learning degree of the cooperators is always larger than that of the defectors, which indicates that, the more learning channels the players possess, the greater is the probability they would cooperate with others.

Finally, we want to point out the difference between our results and those of Ref. [18], in which Ifti *et al.* studied the case where the IN and LN are different in the continuous PD game, and observed that in the lattice topology, when the neighborhood sizes for interacting and learning differ by more than 0.5, cooperation cannot persist in the population. This is not the case for the present studied models wherein the players are pure strategists. Cooperation can be maintained at considerable levels in the cases where the size of the LN is far larger than that of the IN [see Figs. 1(a) and 1(b) for  $d = 3, 4$  and Fig. 2], and can go so far as to wipe out defectors for sufficient small  $r$  (homogeneous state  $C$  in Figs. 1 and 2). In particular, as long as the strategy updating is implemented synchronously, cooperation is always promoted essentially when the IN and LN are different (no matter how large the LN is) as compared when they are identical.

## IV. CONCLUSIONS

In summary, we have explored the influence of the difference between interaction and learning neighborhoods on the evolution of cooperation. This is done by studying an evolutionary spatial PD game wherein the interaction and learning graphs of the players are different. The players are placed on two layered graphs, where the lower layer is the physical infrastructure on which the interactions are taking place and the upper layer represents the skeleton where the payoff and strategy information flow. For the sake of simplicity, we keep the interaction graph fixed and vary the size of the neighborhood in the learning graph. Two types of models have been systematically studied: In model I, we simply increase the size of interaction neighborhood for all the players at the same level; and in model II, we endow the players with heterogeneous ability to obtain information. We performed MC simulations for both models. For model I, we also use an extended pair approximation to evaluate the average density of cooperators,  $\rho_C$ , and make a comparison with the corresponding results that follow from our MC simulations.

The main result is that, a difference between the interaction and learning graphs can promote cooperation substantially. The results of this mechanism resemble a coherence-resonance-like behavior. For model I the maximum enhancement is achieved at  $d = 2$ , i.e., when the players, in addition to their nearest neighbors, can also learn from their next-nearest neighbors; for model II, it is realized at the middle level of the available information of learning neighbors. Too little learning information favors defection, but apparently so does too much information (especially for asynchronous strategy updating). However, as long as the strategy updating is implemented synchronously, cooperation is always promoted essentially when choosing a larger size of neighborhood in the learning graph. This point is also verified by the extended version of the pair-approximation method. In model II, where the players possess heterogeneous learning neighborhoods, we found that the more learning neighbors a player has, the greater the probability it will cooperate with others. There are few existing works studying the evolutionary PD game on networks with distinct interaction and learning neighborhoods. Thus our present results provide a further perspective on understanding the emergence and persistence of cooperation in realistic systems.

In future work, a concise explanation of the mechanism supporting cooperation should be revealed by more sophisticated analytical methods. Furthermore, it would be interesting to allow the interaction neighborhood and/or the learning neighborhood to be mutable during the process of the dynamics (just as has been done in the case of the continuous prisoner's dilemma game [18]), i.e., to study the effects of annealed and quenched randomness in the interaction and/or learning partnership for fixed number of coplayers [20]. Work along these lines is in progress.

## V. ACKNOWLEDGMENTS

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## APPENDIX: EXTENDED PAIR-APPROXIMATION METHOD

When the players are pure strategists, an analytical approximation of the spatial dynamics can be obtained using the pair approximation (a detailed survey of this technique is given in the Appendix of the recent review paper [15], and somewhat brief yet clear versions can be found in the Supplementary Information of [16] and also in the Appendix of [17]). Instead of considering the density of strategies as in well-mixed populations, i.e., in mean-field theory, pair approximation tracks the densities of strategy pairs. For the present studied evolutionary PD games, that is to say, we will first address the probabilities,  $p_{c,c}$ ,  $p_{c,d}$ , of finding an individual playing strategy  $C$  accompanied by a neighbor playing  $C$  or  $D$ , respectively. Then the density of  $C$  is given by  $\rho_C = p_{c,c} + p_{c,d}$ . For more details, we refer the readers to Refs. [15, 16, 17]. Here we just make extensions to the approach to study model I, where the interaction and learning graphs are different. As an example, we will consider the case of  $d = 4$  (extensions to other cases are straightforward).

For  $d = 4$ , each player interacts with its four nearest neighbors on a square lattice, but can learn from those neighbors with longer (Euclidean) distance up to 4. Since these learning neighbors satisfy the condition of rotation symmetry, we will consider only those neighbors falling in the first quadrant

[see Fig. 5]. Whenever a randomly chosen site  $A$  updates its strategy, a neighbor  $B$  is randomly selected from its learning neighborhood as a reference. Their common neighbors (if any) as well as their respective neighbors are considered to be independent by the pair approximation. Thus, when the selected reference is its nearest (next-nearest) neighbor, we will refer to the configuration Fig. 5(b) (Fig. 5(c)); and for other cases we will refer to the configuration Fig. 5(d). Assuming the selected learning neighbor  $B$  is  $A$ 's third neighbor (next-next-nearest neighbor), then we will use the scheme Fig. 5(d) to calculate changes in the pair configuration probabilities  $p_{A,i} \rightarrow p_{B,i}$ .

The payoffs  $P_A$  and  $P_B$  of  $A$  and  $B$  are determined by accumulating the payoffs in interactions with their neighbors  $x, y, z, i$  and  $u, v, w, j$ , respectively. The pair approximation is completed by determining the evolution of the pair configuration probabilities, i.e., the probability that the pair  $p_{A,i}$  becomes  $p_{B,i}$ :

$$p_{A,i \rightarrow B,i} = \sum_{xyz} \sum_{uvw} \sum_{ij} f(P_B - P_A) \times \frac{p_{x,A} p_{y,A} p_{z,A} p_{A,i} p_{j,B} p_{u,B} p_{v,B} p_{w,B}}{p_A^3 p_B^3 p_i p_j}, \quad (\text{A.1})$$

where the transition probability  $f(P_B - P_A)$  [see Eq. (1)] is multiplied by the configuration probability and summed over all possible configurations. If  $B$  succeeds in taking over site  $A$ , the following pair configuration probabilities increase:  $p_{x,B}, p_{y,B}, p_{z,B}, p_{B,i}$ , while the probabilities  $p_{x,A}, p_{y,A}, p_{z,A}, p_{A,i}$  decrease. It is easy to analyze the other cases of  $B$  (i.e., not the third neighbor of  $A$ ), which lead to only a slightly different form of Eq. (A.1). All these changes result in a set of ordinary differential equations:

$$\begin{aligned} \dot{p}_{c,c} = & \sum_{xyz} \{ [n_c(x, y, z) + 1] h_1 + [(n_c(x, y, z) + 1) \frac{p_{c,c}}{p_c} + n_c(x, y, z) \frac{p_{d,d}}{p_d}] h_2 + [(n_c(x, y, z) + 1) (\frac{p_{c,c}^2}{p_c^2} + \frac{p_{c,d}^2}{p_c p_d}) + \\ & n_c(x, y, z) (\frac{p_{d,d}^2}{p_d^2} + \frac{p_{d,c} p_{c,c}}{p_d p_c})] h_3 \} p_{d,x} p_{d,y} p_{d,z} \times \sum_{u,v,w} p_{c,u} p_{c,v} p_{c,w} f(P_c(u, v, w) - P_d(x, y, z)) - \\ & \sum_{xyz} \{ n_c(x, y, z) h_1 + [(n_c(x, y, z) + 1) \frac{p_{c,c}}{p_c} + n_c(x, y, z) \frac{p_{d,d}}{p_d}] h_2 + [(n_c(x, y, z) + 1) (\frac{p_{c,c}^2}{p_c^2} + \frac{p_{c,c} p_{d,d}}{p_c p_d}) + \\ & n_c(x, y, z) (\frac{p_{d,d}^2}{p_d^2} + \frac{p_{c,d}^2}{p_d p_c})] h_3 \} p_{c,x} p_{c,y} p_{c,z} \times \sum_{u,v,w} p_{d,u} p_{d,v} p_{d,w} f(P_d(u, v, w) - P_c(x, y, z)) \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \dot{p}_{c,d} = & \sum_{xyz} \{ [(1 - n_c(x, y, z))] h_1 + [(1 - n_c(x, y, z)) \frac{p_{c,c}}{p_c} + (2 - n_c(x, y, z)) \frac{p_{d,d}}{p_d}] h_2 + [(1 - n_c(x, y, z)) (\frac{p_{c,c}^2}{p_c^2} + \frac{p_{c,d}^2}{p_c p_d}) + \\ & (2 - n_c(x, y, z)) (\frac{p_{d,d}^2}{p_d^2} + \frac{p_{d,c} p_{c,c}}{p_d p_c})] h_3 \} p_{d,x} p_{d,y} p_{d,z} \times \sum_{u,v,w} p_{c,u} p_{c,v} p_{c,w} f(P_c(u, v, w) - P_d(x, y, z)) - \\ & \sum_{xyz} \{ [(2 - n_c(x, y, z))] h_1 + [(1 - n_c(x, y, z)) \frac{p_{c,c}}{p_c} + (2 - n_c(x, y, z)) \frac{p_{d,d}}{p_d}] h_2 + [(1 - n_c(x, y, z)) (\frac{p_{c,c}^2}{p_c^2} + \frac{p_{c,c} p_{d,d}}{p_c p_d}) + \end{aligned}$$

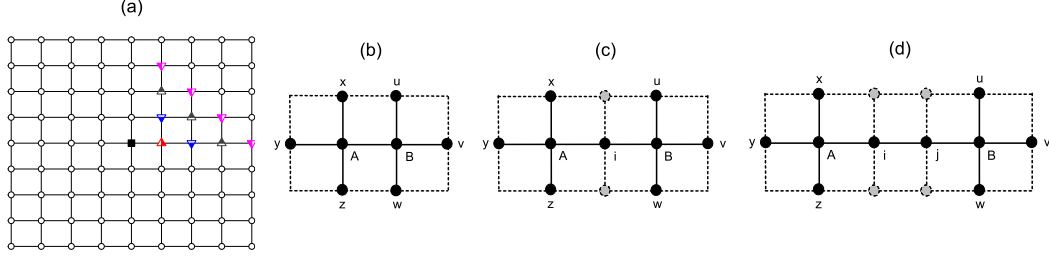


FIG. 5: (Color online) Illustration of the lower interaction graph (a square lattice) and the central site (fully-filled square) has a learning neighborhood of size  $d = 4$  [only shown those neighbors falling in the first quadrant] (a), and the corresponding schemes used for the pair approximation with involved sites  $A, x, y, z, i, j, u, v, w$ , and  $B$  [(b)-(d)]. These schemes are used to determine changes in the pair configuration probabilities  $p_{A,B} \rightarrow p_{B,B}$  (b),  $p_{A,i} \rightarrow p_{B,i}$  (c) and (d).

$$(2 - n_c(x, y, z)) \left( \frac{p_{d,d}^2}{p_d^2} + \frac{p_{c,d}^2}{p_d p_c} \right) h_3 \} p_{c,x} p_{c,y} p_{c,z} \times \sum_{u,v,w} p_{d,u} p_{d,v} p_{d,w} f(P_d(u, v, w) - P_c(x, y, z)), \quad (\text{A.3})$$

TABLE I: The probabilities of selecting the nearest neighbors as references,  $h_1$ , the next-nearest neighbors,  $h_2$ , and the remaining cases,  $h_3$ , for different sizes of the learning neighborhood.

	$h_1$	$h_2$	$h_3$
$d = 1$	1	0	0
$d = 2$	1/3	2/3	0
$d = 3$	1/6	1/3	1/2
$d = 4^a$	1/10	1/5	7/10
$d \rightarrow \infty$	$\approx 0$	$\approx 0$	$\approx 1$

<sup>a</sup>As an example, the values of  $h_1, h_2, h_3$ , for  $d = 4$  can be easily counted out by using the symbolized sites in Fig. 5(a).

where  $n_c(x, y, z)$  is the number of cooperators among the neighbors  $x, y, z$ , and  $P_c(x, y, z)$  and  $P_d(x, y, z)$  specify the payoffs of a cooperator (defector) interacting with the neighbors  $x, y, z$  plus a defector (cooperator).  $h_1, h_2$ , and  $h_3$  denote the probabilities of selecting the first, second, and  $\geq$  third next neighbors as references, respectively (see Table I). For simplicity, the above two equations omit the common factor  $2p_{c,d}/(p_c^3 p_d^3)$  which is inessential [17]. In combination with the symmetry condition  $p_{c,d} = p_{d,c}$  and the constraint  $p_{c,c} + p_{c,d} + p_{d,c} + p_{d,d} = 1$ , the above equations can be treated either by numerical integration or by setting  $\dot{p}_{c,c} = \dot{p}_{c,d} = 0$  and solving for  $p_{c,c}$  and  $p_{c,d}$ . Then the equilibrium density of cooperators is obtained from  $p_c = p_{c,c} + p_{c,d}$ .

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